$$\beta_{CL} = \frac{A_L \bar{x}_L}{EI_L l_L} + \frac{M_L l_L}{6EI_L} + \frac{M_C l_L}{3EI_L} \qquad (13.2a)$$

$$\beta_{CR} = \frac{A_R \bar{x}_R}{EI_R l_R} + \frac{M_R l_R}{6EI_R} + \frac{M_C l_R}{3EI_R} \qquad (13.2b)$$

The rotations of the chord $L'C'$ and $C'R'$ from the original position is given by

$$\alpha_{CL} = \frac{\delta_L - \delta_C}{l_L} \qquad (13.3a)$$

$$\alpha_{CR} = \frac{\delta_R - \delta_C}{l_R} \qquad (13.3b)$$

From Fig. 13.1, one could write,

Equations 13.2(a) , 13.2(b),13.3(a) and 13.3(b) of page 3

Thus, from equations (13.1) and (13.4), one could write,

$$\alpha_{CL} - \beta_{CL} = \beta_{CR} - \alpha_{CR} \qquad (13.5)$$

Substituting the values of $\alpha_{CL}, \alpha_{CR}, \beta_{CL}$ and β_{CR} in the above equation,

$$M_L\left(\frac{l_L}{I_L}\right) + 2M_C\left\{\frac{l_L}{I_L} + \frac{l_R}{I_R}\right\} + M_R\left(\frac{l_R}{I_R}\right) = -\frac{6A_R\bar{x}_R}{I_R l_R} - \frac{6A_L\bar{x}_L}{I_L l_L} + 6E\left(\frac{\delta_L - \delta_C}{l_L}\right) + 6E\left(\frac{\delta_R - \delta_C}{l_R}\right)$$

This may be written as

$$M_L\left(\frac{l_L}{I_L}\right) + 2M_C\left\{\frac{l_L}{I_L} + \frac{l_R}{I_R}\right\} + M_R\left(\frac{l_R}{I_R}\right) = -\frac{6A_R\bar{x}_R}{I_R l_R} - \frac{6A_L\bar{x}_L}{I_L l_L} - 6E\left[\left(\frac{\delta_C - \delta_L}{l_L}\right) + \left(\frac{\delta_C - \delta_R}{l_R}\right)\right]$$

(13.6)

The above equation relates the redundant support moments at three successive spans with the applied loading on the adjacent spans and the support settlements.

Equations 13.5 and 13.6 of page 4

$$\delta_L = \delta_C = 0 \text{ and } \delta_R = -5\times10^{-3}\, m$$

$$M'_A\left(\frac{L'}{\infty}\right) + 2M_A\left\{\frac{L'}{\infty} + \frac{4}{I}\right\} + M_B\left(\frac{4}{I}\right) = -\frac{6\times8\times2}{I(4)} - 6E\left(0 + \frac{0-(-5\times10^{-3})}{4}\right)$$

$$8M_A + 4M_B = -24 - 6EI\times\frac{5\times10^{-3}}{4} \qquad (1)$$

Note that, $EI = 200\times10^9 \times \frac{8\times10^6\times10^{-12}}{10^3} = 1.6\times10^3\,\text{kNm}^2$

Thus,

$$8M_A + 4M_B = -24 - 6\times1.6\times10^3\times\frac{5\times10^{-3}}{4}$$

$$8M_A + 4M_B = -36 \qquad (2)$$

Equations (1) and (2) of page 6

$$M_A\left\{\frac{4}{I}\right\} + 2M_B\left\{\frac{4}{I} + \frac{4}{I}\right\} = -\frac{24}{I} - \frac{6\times10.667\times2}{I\times4} - 6E\left(\frac{-5\times10^{-3}}{4} - \frac{5\times10^{-3}}{4}\right)$$

$$4M_A + 16M_B = -24 - 32 + 6\times1.6\times10^3\times\frac{10\times10^3}{4}$$

$$4M_A + 16M_B = -32 \qquad (3)$$

Equation (3) of page 6

A continuous beam $ABCD$ is supported on springs at supports B and C as shown in Fig.13.3a. The loading is also shown in the figure. The stiffness of springs is $k_B = \frac{EI}{20}$ and $k_C = \frac{EI}{30}$ .Evaluate support reactions and draw bending moment diagram. Assume EI to be constant.

Example 2 of page 8

Now applying three moment equations to span ABC (see Fig.13.2b)

$$M_A\left\{\frac{4}{I}\right\}+2M_B\left\{\frac{4}{I}+\frac{4}{I}\right\}+M_C\left\{\frac{4}{I}\right\}=-\frac{6\times21.33\times2}{I\times4}-\frac{6\times20\times2}{I\times4}-6E\left[\frac{-20R_B}{4EI}+\frac{\frac{-20R_B}{EI}+\frac{30R_C}{EI}}{4}\right]$$

Simplifying,

$$16M_B+4M_C=-124+60R_B-45R_C \qquad (2)$$

Again applying three moment equation to adjacent spans BC and CD,

$$M_B\left\{\frac{4}{I}\right\}+2M_C\left\{\frac{4}{I}+\frac{4}{I}\right\}=-\frac{60}{I}-\frac{(6\times9\times2+6\times3\times\frac{2}{3}\times1)}{I\times4}-6E\left[\frac{-\frac{30R_C}{EI}+\frac{20R_B}{EI}}{4}+\frac{-30R_C}{4EI}\right]$$

$$4M_B+16M_C=-90+90R_C-30R_B \qquad (3)$$

Equations (2)& (3) of page 10

$$\theta_A=\beta_{AR}-\alpha_{AR}$$

$$=\frac{A_R\bar{x}_R}{EI_Rl_R}+\frac{M_Bl_R}{6EI_R}+\frac{M_Al_R}{3EI_R}-\left(\frac{\delta_B-\delta_A}{4}\right)$$

$$=\frac{6\times8\times2}{1.6\times10^3\times4}+\frac{M_B\times4}{1.6\times10^3\times6}+\frac{M_A\times4}{1.6\times10^3\times3}-\left(\frac{\delta_B-\delta_A}{4}\right)$$

$$=\frac{6\times8\times2}{1.6\times10^3\times4}+\frac{(-1)\times4}{1.6\times10^3\times6}+\frac{(-4)\times4}{1.6\times10^3\times3}+\left(\frac{5\times10^{-3}}{4}\right)$$

$$=0 \qquad (1)$$

For calculating θ_{BL}, consider span ABC

$$\theta_{BL}=\alpha_{BL}-\beta_{BL}$$

$$=-\left(\frac{A_L\bar{x}_L}{EI_Ll_L}+\frac{M_Al_L}{6EI_L}+\frac{M_Bl_L}{3EI_L}\right)+\left(\frac{\delta_A-\delta_B}{l_L}\right)$$

$$=-\left(\frac{8\times2}{1.6\times10^3\times4}+\frac{(-4)\times4}{1.6\times10^3\times6}+\frac{(-1)\times4}{1.6\times10^3\times3}\right)+\left(\frac{5\times10^3}{4}\right)$$

$$=1.25\times10^{-3}\text{ radians} \qquad (2)$$

For θ_{BR} consider span ABC

Equations (1) & (2) of page 13

$$\theta_{BR} = \left(\frac{10.67 \times 2}{1.6 \times 10^3 \times 4} + \frac{(-1) \times 4}{1.6 \times 10^3 \times 3} \right) - \left(0 + \frac{5 \times 10^3}{4} \right)$$

$$= -1.25 \times 10^{-3} \text{ radians} \qquad (3)$$

$$\theta_C = -\left(\frac{10.67 \times 2}{1.6 \times 10^3 \times 4} + \frac{(-1) \times 4}{1.6 \times 10^3 \times 3} \right) - \left(\frac{\delta_B - \delta_C}{4} \right)$$

$$= -3.75 \times 10^{-3} \text{ radians.} \qquad (4)$$

The deflected shape of the beam is shown in Fig. 13.4.

Equations (3) & (4) of page 14